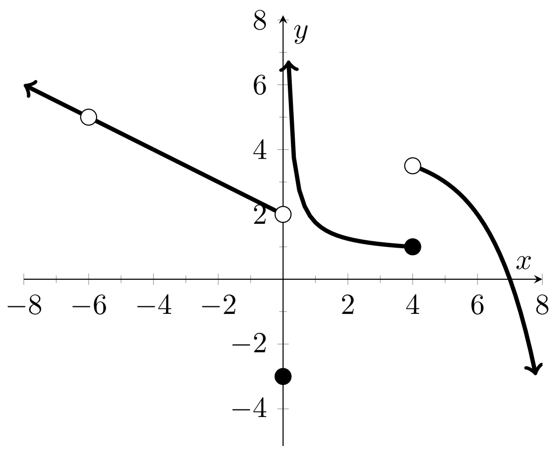
Math 251 Fall 2017 **Quiz #3, September 20**

Name: Solutions

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (5 pts.) Consider the function $f(x)$ with graph given below.



a.) List any values a where $\lim_{x\to a} f(x)$ fails to exist.

0, 4

b.) List any values x where $f(x)$ fails to be continuous. Describe the type of discontinuity at each such value a.

0 is an infinite discontinuity
−6 is removable and
4 is a jump discontinuity.

Exercise 2. (4 pts.) Evaluate $\lim_{x\to 3} \dfrac{x^2-5x+6}{3-x}$.

$$\lim_{x\to 3} \frac{x^2-5x+6}{3-x} = \lim_{x\to 3} \frac{(x-3)(x-2)}{3-x} = \lim_{x\to 3} -(x-2) = -1.$$

Exercise 3. (4 pts.) Evaluate $\lim_{x\to 2} \dfrac{\frac{1}{4}-\frac{1}{2x}}{x-2}$.

$$\lim_{x\to 2} \frac{\frac{1}{4}-\frac{1}{2x}}{x-2} = \lim_{x\to 2} \frac{\frac{x-2}{4x}}{x-2} = \lim_{x\to 2} \frac{1}{4x} = \frac{\lim_{x\to 2} 1}{\lim_{x\to 2} 4x} = \frac{1}{8}$$

Exercise 4. (5 pts.) Consider the function

$$f(x) = \begin{cases} \frac{2}{x} & x < 2 \\ 3 & x = 2 \\ 3 - x & x > 2 \end{cases}$$

a.) Evaluate $\lim_{x\to 2} f(x)$.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2/x = \frac{\lim_{x\to 2^-} 2}{\lim_{x\to 2^-} x} = \frac{2}{2} = 1$$ and

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3-x) = 3 - 2 = 1.$$ Since they agree,

$$\lim_{x\to 2} f(x) = 2.$$

b.) Explain why $f(x)$ fails to be continuous at $x = 2$.

$$\lim_{x\to 2} f(x) = 2 \neq 3 = f(2).$$

Exercise 5. (4 pts.) Using complete sentences, explain why the function $f(x) = x^2 - 4 + \sin x$ has a zero on the interval $[-\pi, 0]$.

Observe that $f(-\pi) = (-\pi)^2 - 4 + 0 = \pi^2 - 4 > 0$ and $f(0) = 0^2 - 4 + 0 < 0$ and that $f(x)$ is continuous on $[-\pi, 0]$. So by the Intermediate Value theorem, there is $-\pi < c < 0$ such that $f(c) = 0$.

Exercise 6. (3 pts.) If $-x^4 + x^2 - 1 \leq g(x) \leq -x^2$ for all x, evaluate $\lim_{x\to 1} g(x)$. Justify your answer.

Observe that $\lim_{x\to 1} (-x^4 + x^2 - 1) = -1 + 1 - 1 = -1$

and $\lim_{x\to 1} (-x^2) = -1$ so by the Squeeze theorem

$$\lim_{x\to 1} g(x) = -1.$$